Mortgage structure, saving rates and the wealth distribution

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Introduction

How do fixed amortization schedules in mortgages affect homeowners' saving and (the distribution of) wealth?

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Mortgage *debt* contracts are a large *saving* plan

- Homeowners: $\sim 60\%$ of saving is mortgage repayment in the Euro area (similar in US)
  - → $\rightarrow \sim 40\%$ of the population
- In many countries (Euro area, US), only available structure is a **fully amortizing annuity loan**:
  - → Fixed payment = interest + principal. Balance $\rightarrow 0$ at maturity
- Repayment schedule fixed at origination and costly to deviate from (refinancing, late penalties, …)

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Mandatory amortization schedule $\Rightarrow \uparrow$ saving, $\downarrow$ consumption

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This paper. A theory of consumption/saving under different mortgage structures suggests:

- This can be **rationalized** by standard model w/ costly deviation from repayment schedule
- It may have **large, heterogeneous effects** on saving over the life cycle $\rightarrow$ wealth distribution

This paper

Life cycle model of homeowners facing uninsurable income risk and a fixed amortization schedule

- Explains large effects on consumption in empirical literature $\rightarrow \uparrow\uparrow$ saving rate
  - $\rightarrow$ Effects are heterogeneous: younger, poorer homeowners save more; others unaffected

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Matches **novel stylized facts** from household wealth data in Europe

- Younger and lower-income/wealth homeowners with an amortizing mortgage save more
  - → Homeowners 30-40y.o. in Europe save 2x more than renters/free users
- Homeowners with *interest-only* mortgages in **Netherlands** similar to renters
  - → No differences among older, richer groups

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  - $\rightarrow$ No differences among older, richer groups

Large implications of mandatory amortization for wealth accumulation & distribution

- $\uparrow$ Saving rates for young and lower-income homeowners – but leaves them more exposed to shocks:
  - $\rightarrow$ $\uparrow$ Total wealth/income ratios, but $\downarrow$ liquid wealth $\Rightarrow$ higher $\%$ HtM, MPCs, C volatility

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Contribution to the literature

- Effects of mortgage amortization on household consumption and saving

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- Optimal mortgage payment structure

  Boar et al. (2022); Balke et al. (2024), Boutros et al. (2025); Campbell and Cocco (2015), Campbell et al. (2018), Chambers et al. (2009), Greenwald et al. (2018), Guren et al. (2018), Piskorski and Tchistyi (2010, 2011)

  → This paper: (heterogeneous) effects on household wealth and welfare of repayment rigidity

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- Wealth distribution: housing drives dynamics through return rates

  Saez & Zucman (2016); Jorda et al. (2019), Fagereng et al. (2020), Kuhn, Schularick & Steins (2020); Martinez-Toledano (2022)

  → This paper: role of saving rates channel due to mortgage contract design

Agenda

1. Introduction
2. Model framework and insights
3. Data: stylized facts and calibration
4. Model results
5. Conclusion

Model framework and insights

Model framework

Overview

Standard incomplete markets model + mortgage debt

- First-time homebuyer life-cycle
  - → From origination to maturity of the mortgage
- Basic features:
  - → Two asset types: **liquid safe asset** (risk-free) vs. **mortgage debt**. Housing fixed
  - → Idiosyncratic income risk (permanent + transitory)
- Key addition: mortgage contract transaction costs
  - → Mandatory amortization schedule: cost to delay repayment
  - → How does this wedge affect saving and wealth accumulation?

Model

Household life cycle endowments and decisions

- A home worth P_0 (normalized) and a 30-year fixed-rate mortgage with initial balance M_0
- Some initial financial wealth: A_0 and exogenous risky earnings Y_t over the life cycle
- Decide each period on how much to:
  - → consume c_t and save each period
  - → repay d_t of their mortgage debt

Households in the model maximise utility from non-housing consumption:

$$U(c_t) = \frac{{c_t}^{1-\gamma}}{1-\gamma}$$

- Only non-housing consumption enters utility (housing H fixed)
  - → Assumption: prefs separable, so $\mathbf{argmax} \sum_t u(C_t) = \mathbf{argmax} \sum_t u(C_t, \bar{H})$ (Campbell-Cocco 2015)

Model framework

Assets & mortgage frictions

Liquid saving and mortgage debt

- Savings in the liquid asset (a_t) earn risk-free interest
  - → Borrowing limit $a_t \geq 0$ (no unsecured debt)
  - → Household cannot increase mortgage debt, only repay $d_t \geq 0$
- Outstanding mortgage debt demands interest $r + s$

Mortgage repayment schedule

- Mandatory amortization: $D^*(m_{t-1}, t)$ from standard annuity formula
  - → Deviating from repayment schedule $d_t < d_t^*$, then incurs transaction cost $\tau_t \geq 0$
- If default, lose house and keep low consumption $\underline{c}$ until end
  - → Repayment usually feasible under calibration $y : y > D^*(m_{t-1}, t) + m_{t-1}(r + s)$

Model insights

Period problem

$$\max_{c_t, d_t} \; u(c_t) \; + \; \beta \, \mathbb{E}_t \left[V_{t+1}(y_{t+1}, a_{t+1}, m_{t+1}) \right]$$

$$a_{t+1} = (1 + r)\left[a_t + y_t - (r + s) m_t - d_t - \tau_t - c_t \right]$$

$$m_{t+1} = m_t - d_t \qquad\qquad m_t \geq 0, \; a_t \geq 0$$

- Key friction: scheduled repayment d_t^*, underpaying costs $\tau_t \equiv \tau \cdot \max\{0, d_t^* - d_t\}$

FOC for amortization trades-off marginal value of liquid asset accumulation vs. mortgage repayment

Model insights

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FOC for amortization trades-off marginal value of liquid asset accumulation vs. mortgage repayment

- For some states (a, y, m), without the cost of delaying ($\tau = 0$), $d_t < d_t^*$ preferable:

$$(1 + r - \tau)\mathbb{E}_t[V_a'] < \mathbb{E}_t[V_m'] < (1+r)\mathbb{E}_t[V_a']$$

- τ introduces wedge: if liquid assets/income low, but not *too much*, HH sticks to d_t^* and reduces c_t, a_{t+1}
  - → If $\tau = 0$, HH would prefer to delay repayment and increase c_t, a_{t+1}
- Far from liq. constraint, $\mathbb{E}_t[V_a']$ is lower so τ irrelevant (as $s > 0$)

Model insights

Mechanism: how amortization frictions affect saving

Predictions for consumption and saving under mandatory amortization

- Stronger effects for:
  - → Younger: higher expected income growth, lower income, lower wealth (life cycle; down payment)
  - → Lower-income: houses, mortgages indivisible
- Little or no effect for wealthier or higher-income homeowners
- Compared to:
  - → Flexible repayment scheme (e.g. *interest-only mortgages*)
  - → Renters and others
- Consequence: higher saving rates for constrained mortgaged homeowners
  - → Matches stylized facts in Euro area data → life-cycle and income/wealth saving gradients

Data: stylized facts and calibration

Data from euro area countries, focus on NL

The Eurosystem HFCS - Household Finance and Consumption Survey

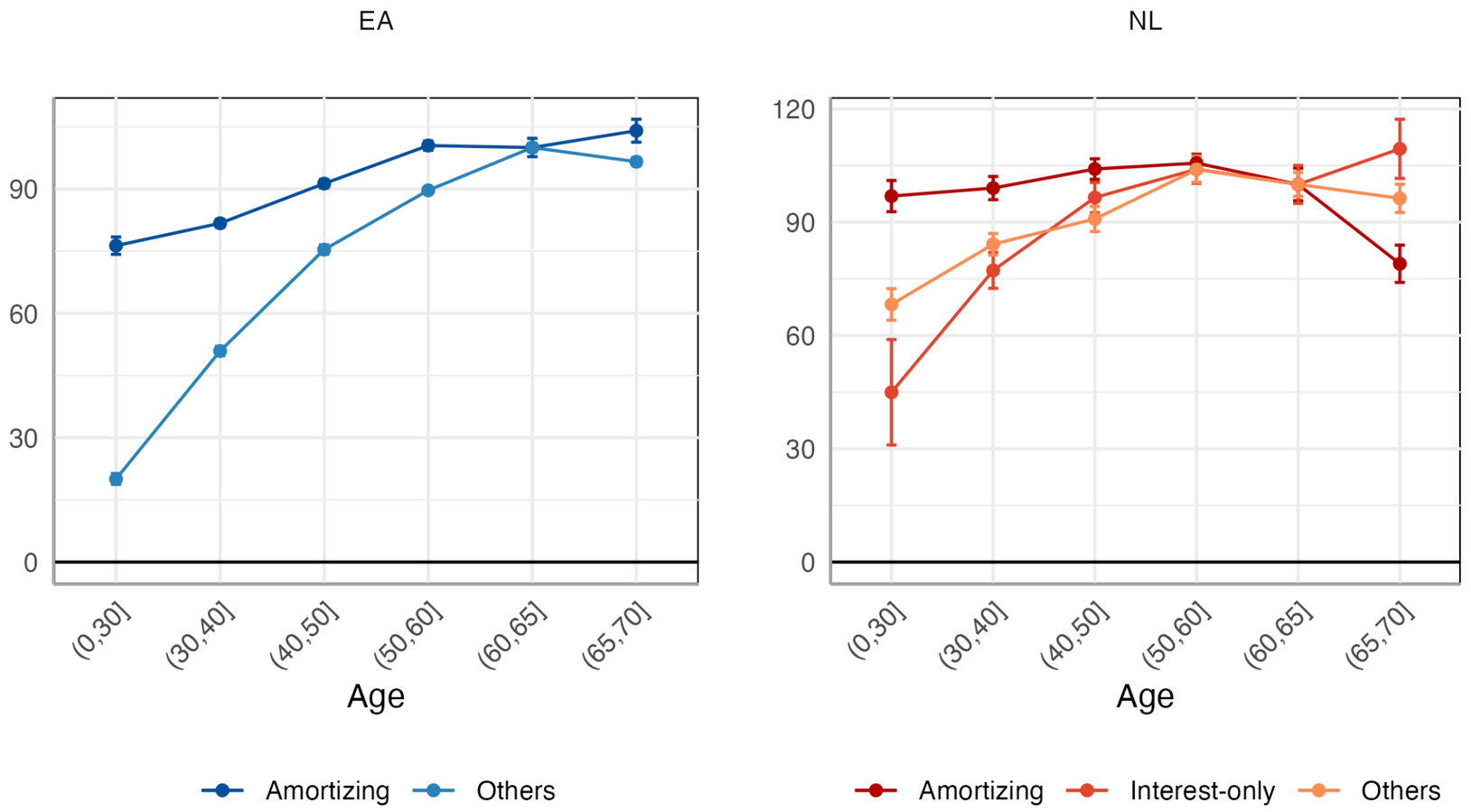
- Harmonized survey of households in Euro area. Three waves (2013-14; 2016-17; 2020-21)
- Compare avg. of Euro area versus Netherlands (NL): **mostly interest-only mortgages**
- Netherlands policy reform in 2013:
  - → From 2013, MID restricted to **fully amortizing loans** – high cost of deferred payment
  - → New borrowers forced to amortize → sharp rise in repayment flows

- Data on saving rates from consumption and net income
- **Amortization** backed out from regular payment: $12 \times \mathbf{mthly\ pmt}_i - i_i \times \mathbf{debt}_i$, for HH i
  - → The median household in EA amortizes ~10% of yearly income; 2.5% in NL Amortization histograms
  - → Various checks on amortization measure Regular payment / income

    Histogram by waves | Annuity formula | Interest rates
- Exclude elderly/retired: $\mathbf{Age} > 70$

Amortizing mortgages increase saving at the beginning of life cycle

Saving rates over the life cycle (Age 65 = 100)

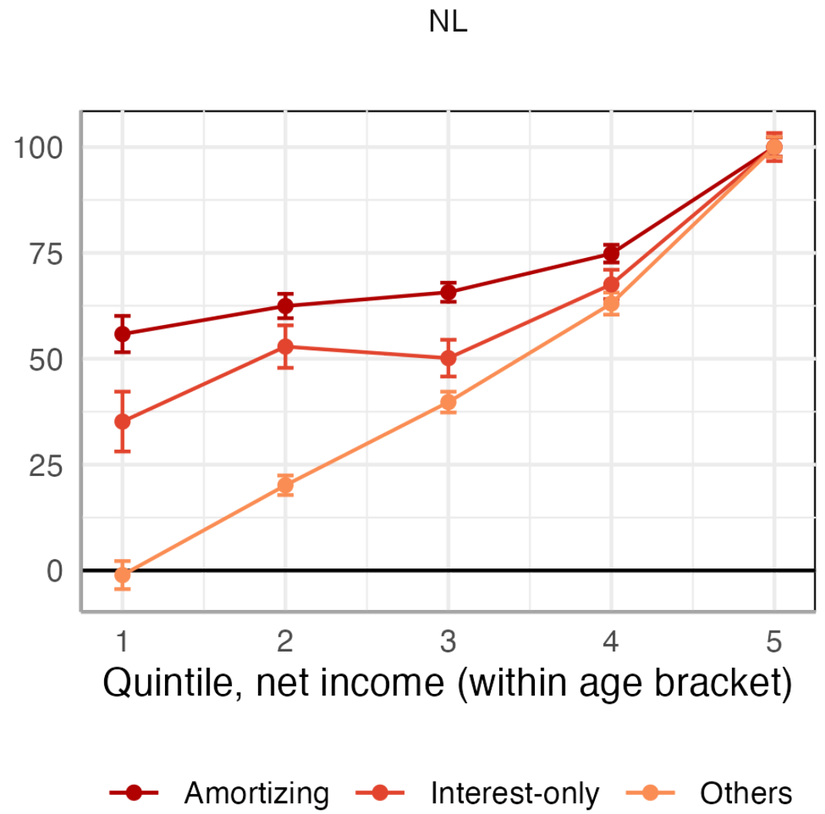


- **Interest-only mortgages** show pattern of renters/outright owners

Post-2013 policy | Life cycle profiles of assets and debt

Amortizing mortgage increase saving only for poorer homeowners

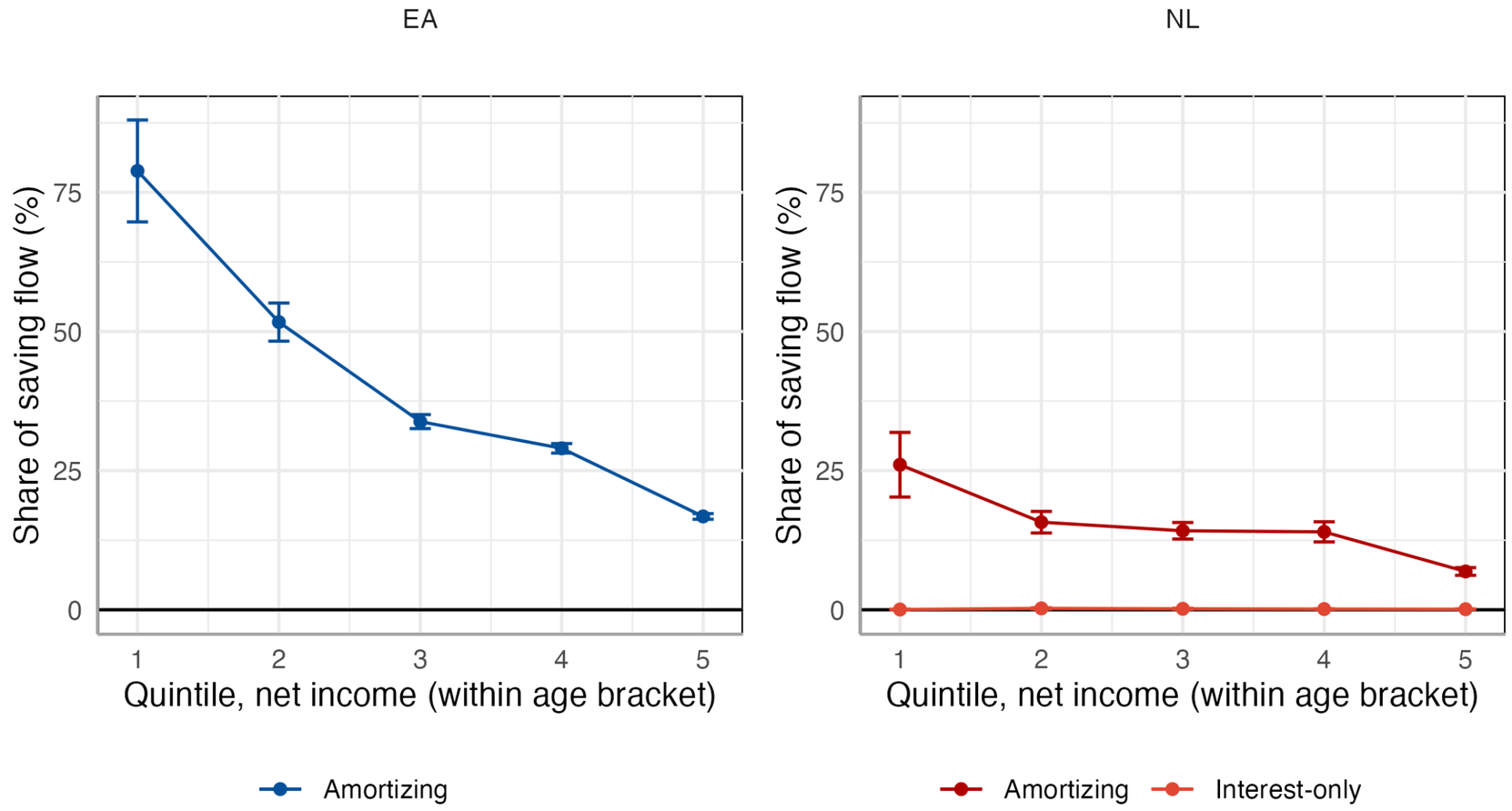
Saving rates over the income distribution (Q5 = 100)



- Again IO mortgages show pattern of renters/outright owners Post-2013 policy
- Age + income heterogeneity → same patterns over the wealth dist Saving rates over wealth dist

% of saving going to amortization declines with income in EA, less in NL

Amortization as % of saving flow



- In EA without interest-only, % of saving to amortization very high for more constrained homeowners

Calibration

Calibration strategy

1. **Fixed/externally calibrated params from data**
   - Income process: life-cycle profiles + stochastic properties - 2 education types
   - Initial conditions: empirical distributions of wealth, debt, house value
   - Interest rates (fixed + variable), potentially risky
   - Preferences off-the-shelf
2. **3 estimated parameters**
   - β: discount factor
   - b_0: bequest motive strength
   - τ^{pre} pre-2013 cost of delaying repayment
   - τ^{post} post-2013 cost of delaying repayment
3. **Target moments: 6 moments from HFCS** (x 2 types)
   - Post-2013 and pre-2013 early repayment
   - Pre-2013 late repayment and liquid wealth

Model framework

Full dynamic household problem

In practice, solved in terms of consumption c_t and a transformed repayment share ψ_t, where:

$$\psi_t \equiv \frac{d_t}{y_t - (r+s)m_t - \tau_t - c_t} \quad \text{(share of saving used for mortgage repayment)}$$

The household solves the dynamic problem:

$$V(t, s_t) = \max_{\{c_k, \psi_k\}_{k=t}^{T}} \mathbb{E}_t \left[\sum_{k=t}^{T-1} \beta^{k-t} \frac{c_k^{1-\gamma}}{1-\gamma} + \beta^{T-t} B(a_T - m_T) \right], \text{ s.t.}$$

$$\begin{aligned} d_t &= \psi_t \cdot (y_t - (r+s)m_t - \tau_t - c_t) \\ a_{t+1} &= (1+r)\left[a_t + y_t - (r+s)m_t - d_t - \tau_t - c_t\right] \\ m_{t+1} &= m_t - d_t \\ \tau_t &= \tau \cdot \max\{0, d_t^* - d_t\}, \quad a_t \geq 0, \quad m_t \geq 0, \quad d_t \geq 0 \end{aligned}$$

- Solution: deep learning algorithm proposed by Duarte et al. (2022), Barrera & Silva (2024)

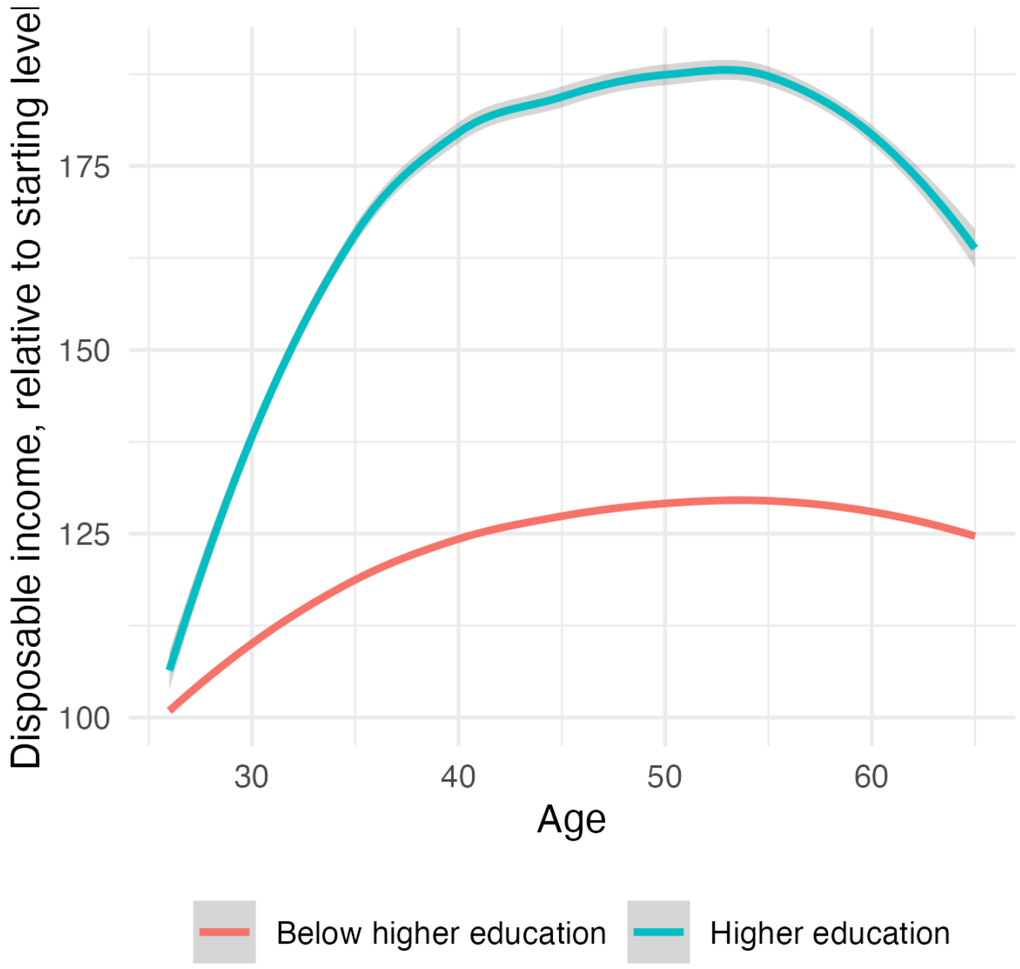
Calibration

Income process

Inelastic labor supply yields earnings $Y_t = \Gamma_t Z_t \theta_t$, as standard (Carroll & Samwick, 1997)

- $\ln Z_t = \ln Z_{t-1} + \ln \psi_t$;
  $\ln \psi_t \sim N\left(-\frac{1}{2}\sigma_\psi^2, \sigma_\psi^2\right)$; $\ln \theta_t \sim N\left(-\frac{1}{2}\sigma_\theta^2, \sigma_\theta^2\right)$
- Life-cycle profile Γ from HFCS
- Moments of stochastic process from NL micro data

  (de Nardi et al. 2021)
- Two types: college vs. lower education
  - → Different income levels and income growth patterns
  - → Different price/income ratios (but LtVs same)



Note: 4th-deg polynomial fit

Calibration

Initial conditions drawn from empirical distributions

- Source: Netherlands HFCS, mortgage holders with ≤ 2 years since origination (purchase $\approx$ observation)
  - → Working on extending with DNB data
- Separate datasets by education type

Households simulated from empirical distribution:

- Purchase age (25–40)
- House value / income (P_0 / Y_0)
- Loan-to-value (M_0 / P_0)
- Liquid wealth / income (A_0 / Y_0)
- **Sample each obs. so the model inherits the observed correlations directly**

Features:

- Older buyers tend to have higher liquid wealth; wealthier buyers select lower LTVs

Calibration

Terminal conditions: *bequest motive at retirement* to match end-of-life wealth and mortgage debt:

$$B\big(a_T - m_T\big) = \underline{b}, \; \frac{\big(a_T - m_T + \overline{b}\big)^{\,1-\gamma}}{1-\gamma}, \quad \underline{b}, \overline{b} \text{ params}$$

- Mortgage must be fully repaid by retirement $\Leftrightarrow$ bequest is net wealth $a_T - m_T$
- Parameters $\underline{b}, \overline{b}$ (set $\underline{b} = 0$, calibrate $b \equiv \overline{b}$)

Calibration strategy

Four preference and transaction-cost parameters estimated to match lifecycle patterns:

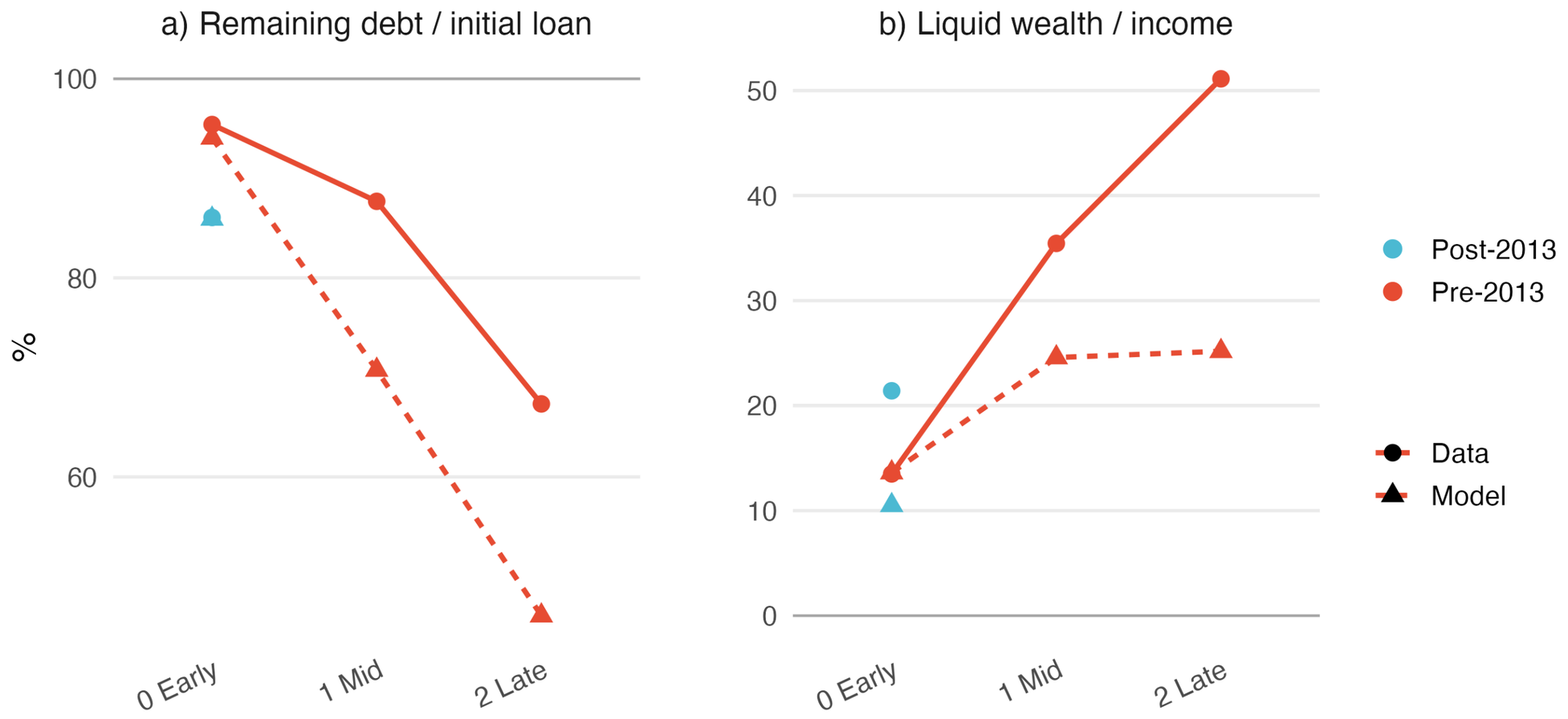
Parameter	Description	Identifies
β	Discount factor	Overall wealth accumulation over life cycle
b_0	Bequest strength	Terminal wealth before retirement
τ^{pre}	Pre-2013 repayment friction	Baseline repayment dynamics
τ^{post}	Post-2013 repayment friction	Policy effect on repayment

Ten target moments from HFCS spanning early and late mortgage lifecycle:

Regime	Mortgage age	Moments (by education type)
Post-2013	Early (2-5 yrs)	Liquid wealth/income (2); % debt outstanding (2)
Pre-2013	Early (2-5 yrs)	% debt outstanding (2)
Pre-2013	Late (25-30 yrs)	Liquid wealth/income (2); % debt outstanding (2)

Interest rates and liquid returns externally calibrated

Model matches key targeted moments



- Key policy effect replicated: post-2013 debt falls faster despite lower interest rates
- Lifecycle wealth accumulation captured for both education groups

Calibrated parameters

Estimated parameters are reasonable

	Estimate	Interpretation
β	**0.97**	Standard discount factor (annual patience)
b_0	**1.2**	Value of liquid wealth post-retirement not especially high
τ^{pre}	**0.37**	Small cost of delaying repayment pre-2013 (flexible repayment)
τ^{post}	**1.06**	**Strong post-2013 friction: 106% penalty for delayed repayment**

Policy effect significant

$\Delta\tau = 0.69$ **represents the effect of mandatory amortization policy**

- Reflects combination of tax penalty and change in product offering by banks
- Large enough to overcome negative spread (post-2013 rates fell below liquid returns)
- Consistent with sharp observed increase in early-stage repayment

Calibration

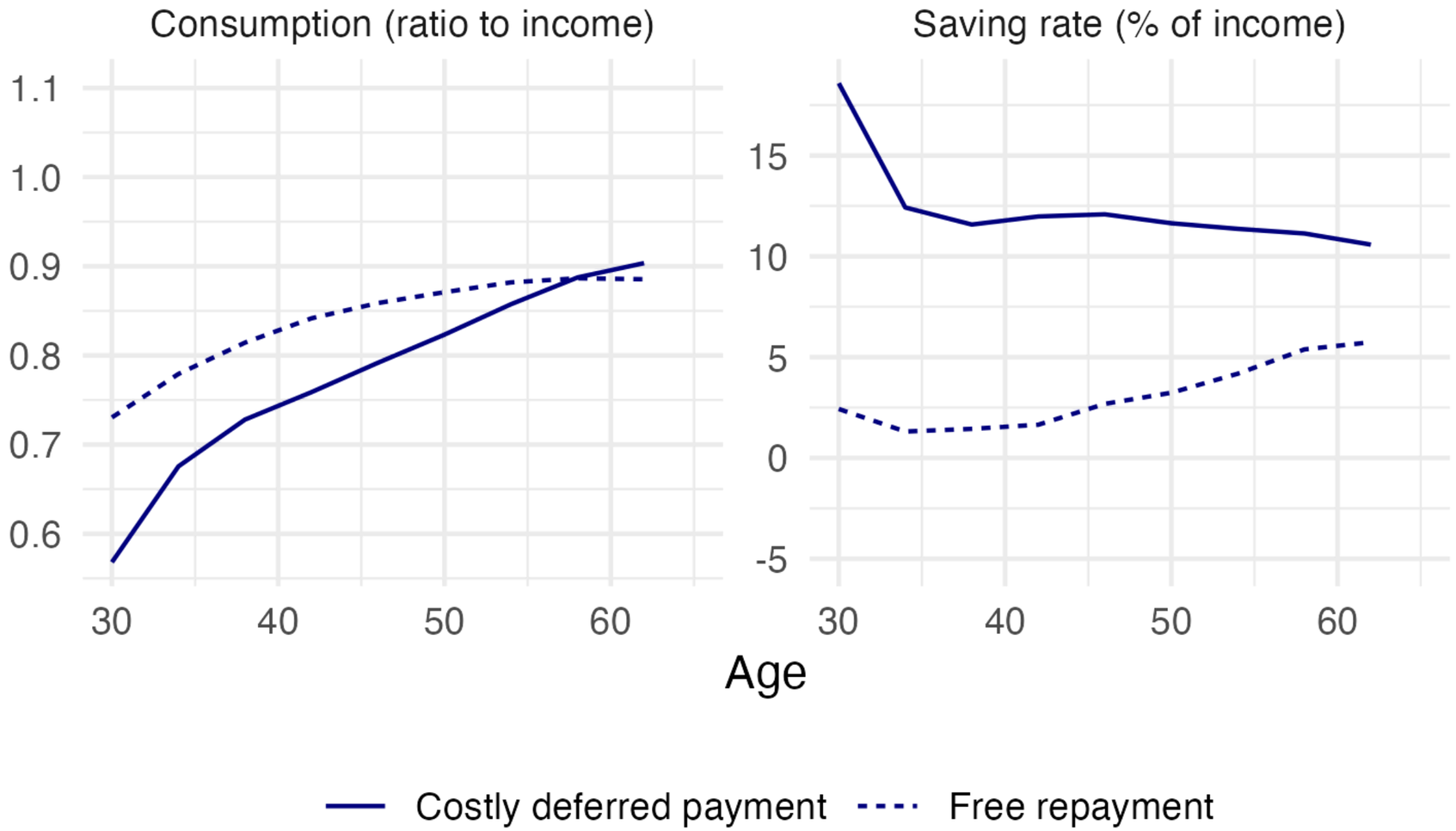
Externally calibrated parameters

Description	Value	Notes	Source
Life time in the model (T)	30	Most common maturity	HFCS and Hypostat (2019)
Risk aversion coeff., consumption (γ)	5	-	Duarte et al. (2020)
Bequest motive parameters (b)	1.5	Wealth at retirement	HFCS 2017 micro data
Risk aversion coeff., bequest (γ^b)	2	-	–
Permanent income life cycle path	-	-	HFCS 2017 micro data
Persistence of permanent shocks	-	Age-varying	Paz-Pardo et al (2020)
Variance of transitory shocks (σ_y^2)	0.015	Earnings shocks (transitory)	Paz-Pardo et al (2020)
Variance of permanent shocks (σ_z^2)	0.01	Earnings shocks (permanent)	Paz-Pardo et al (2020)
Riskless rate (r)	0.02	Long-run real safe rate	Jordà et al. (2019)
Mortgage spread	0.014	NL median fixed-rate spread, adjusted for tax wedge	HFCS 2017 micro data
Borrowing limit, liquid (θ^A)	-0.5	Share of income	-
Borrowing limit, mortgage LTV (θ^M)	120%	Maximum common value in data	DHS, HFCS

Model results

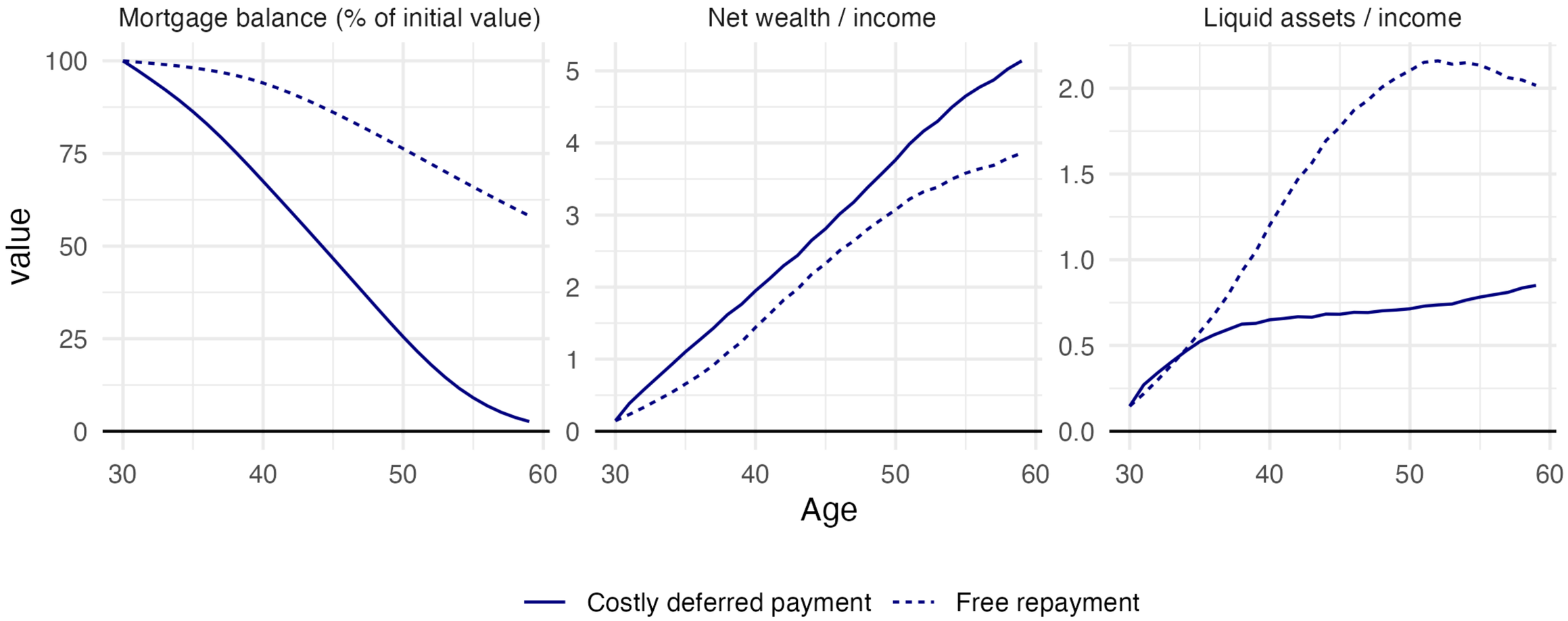
Model HHs forced to amortize cut consumption until mortgage is repaid

Average age profiles of consumption and saving



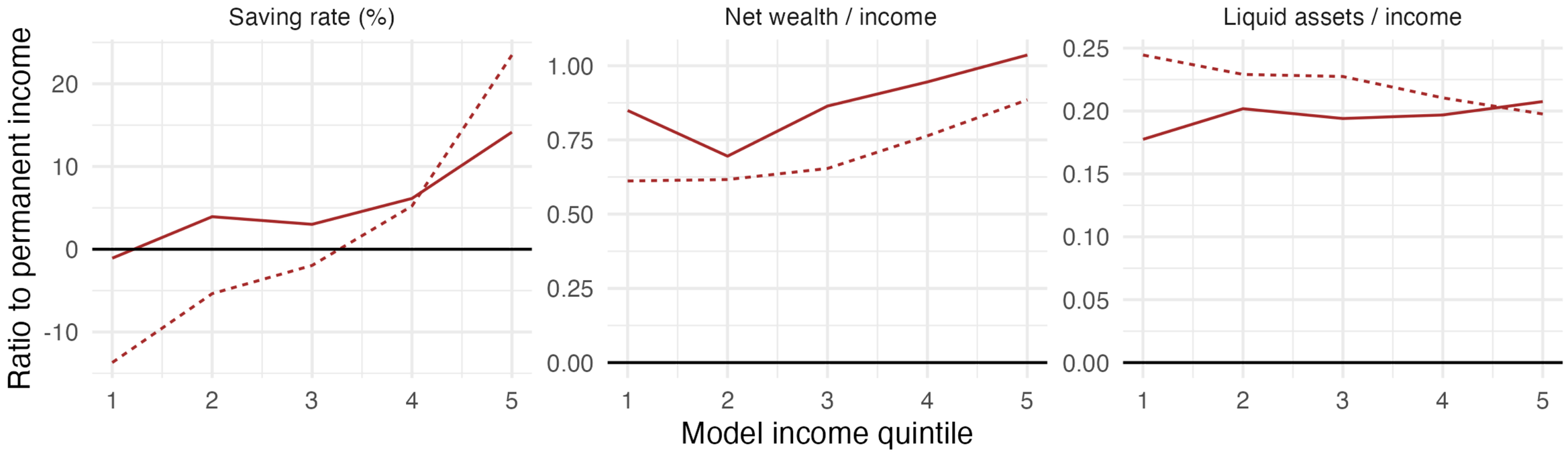
Model HHs allowed to optimize backload repayment

Average age profiles of mortgage balance and wealth



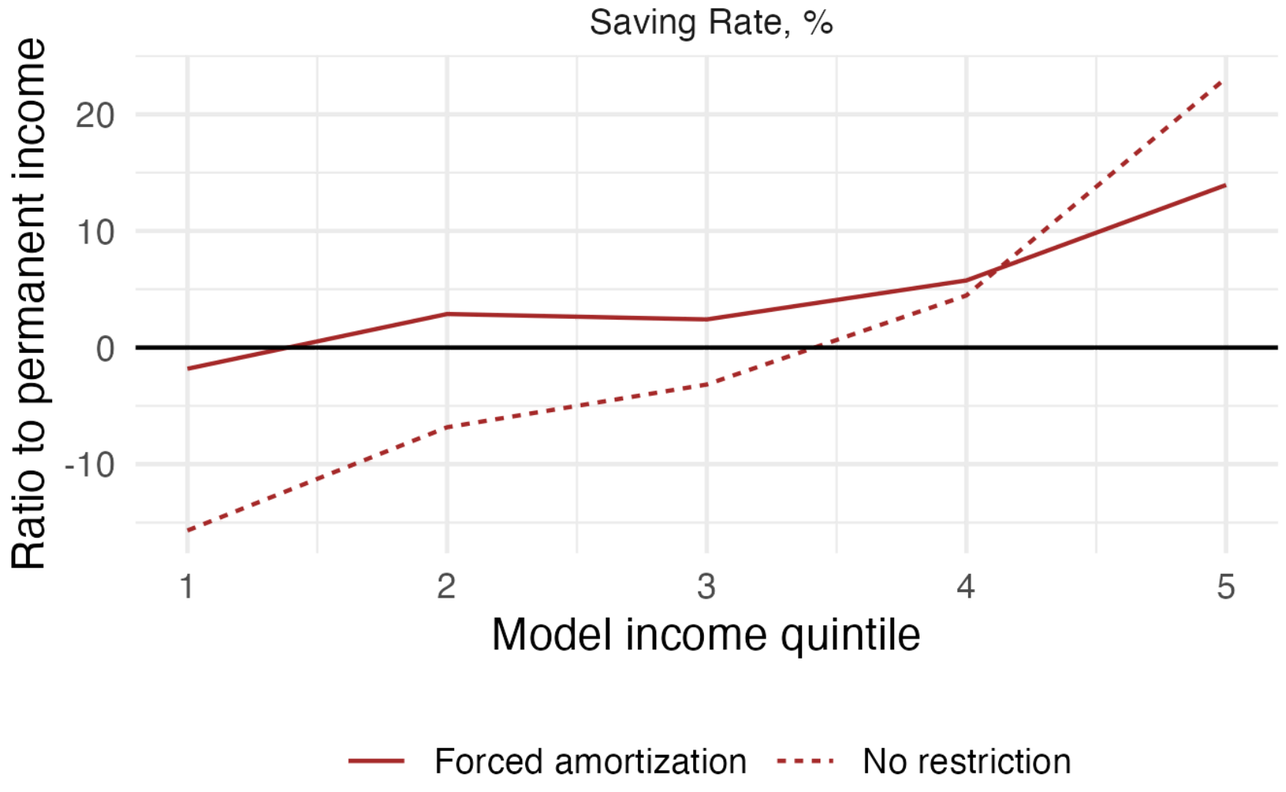
Income-poorer model HHs save more in total, but less into liquid wealth

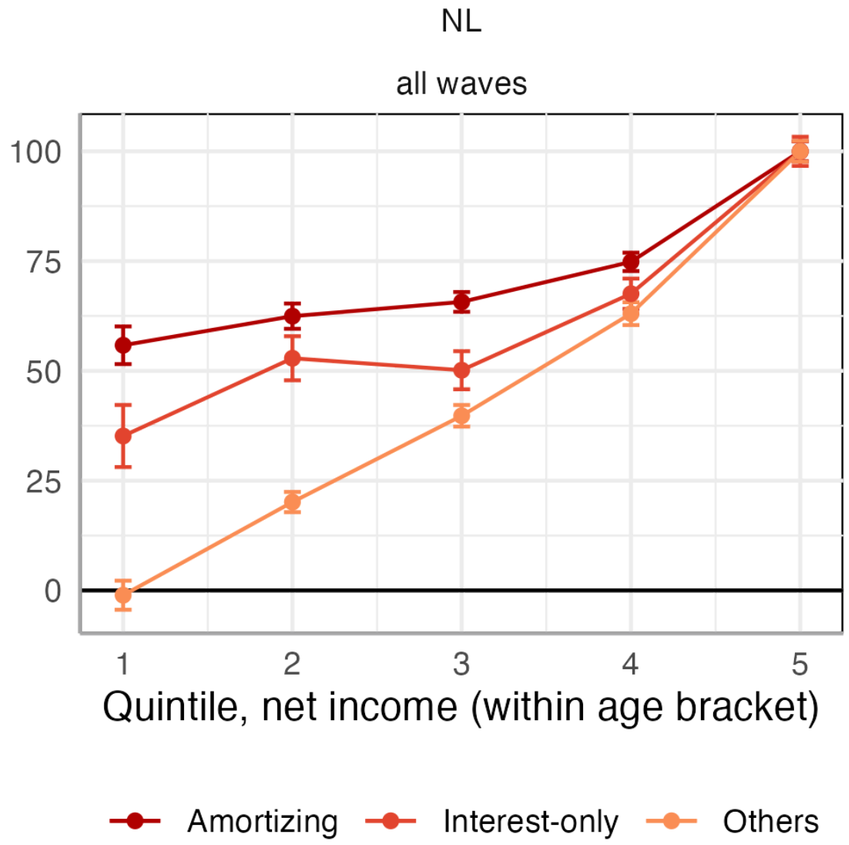
Means of model population across income quintiles (conditional on age)



- Saving rate increases, but $\downarrow C$, liquid savings
- More exposed to shocks $\Rightarrow$ higher MPCs, C volatility

Flattening of saving rate differences reproduces pattern in the data

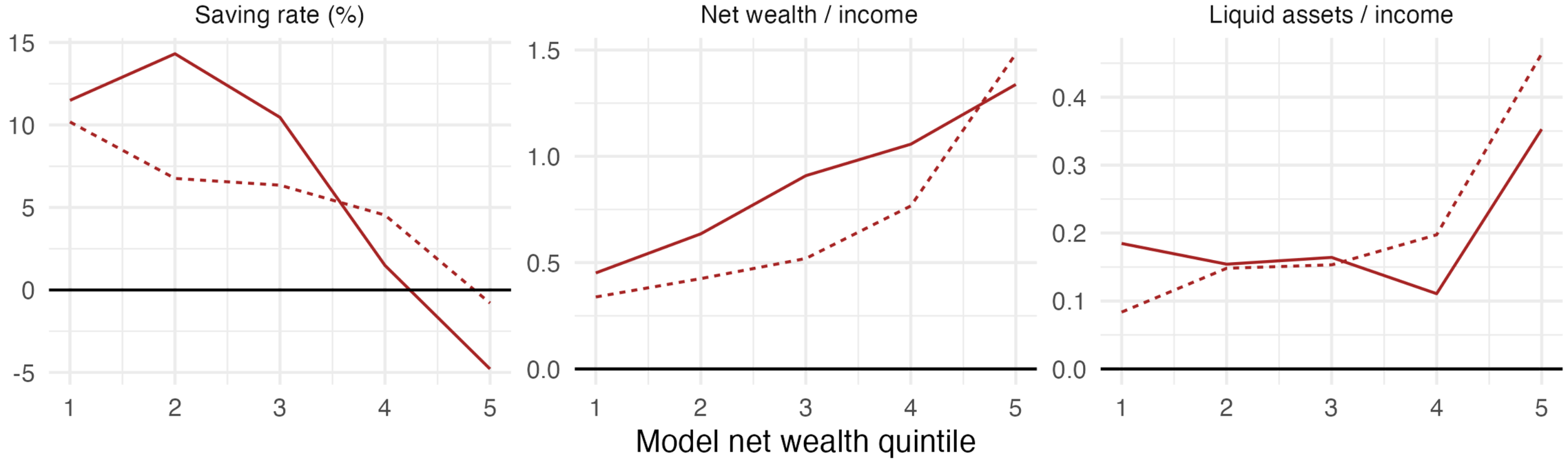




- Saving rates increase for lower income (and younger ages)

Forced amortization increases saving rates at the bottom of wealth dist.

Suggesting effects on distribution of total and financial wealth



- Saving rates increase for groups at the bottom wealth groups
- Implications for wealth distribution: ↓ total wealth inequality but ↑ financial w. ineq., $\%HtM$

Conclusion

Conclusion

- Mortgage debt repayment is an important part of household saving flows
- **Precautionary saving response of homeowners** in standard model rationalizes:
  - → Reduced-form lit: large effects of mandatory amortization on C
  - → Stylized facts in Europe: young, low-income homeowners save more; richer unaffected
- Important implications for consumption and wealth distribution:
  - → $\uparrow$ Total wealth/income ratios, but $\downarrow$ liquid wealth $\Rightarrow$ higher $\%$ HtM, MPCs, C volatility
  - → Financial stability benefits must be weighed against costs for households
  - → Younger, lower-income households seem to be unduly penalized

Thank you!

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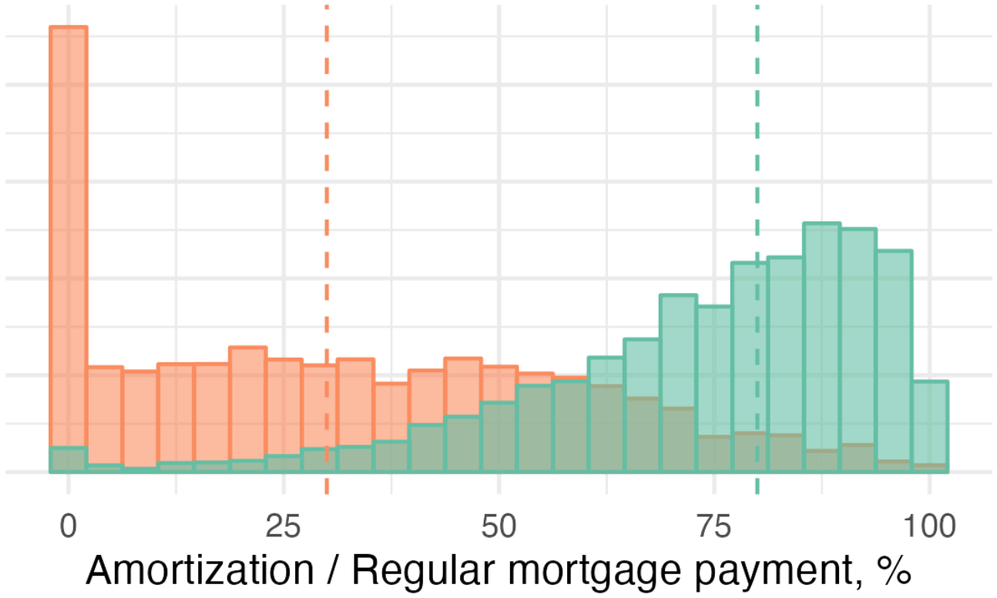
Appendix

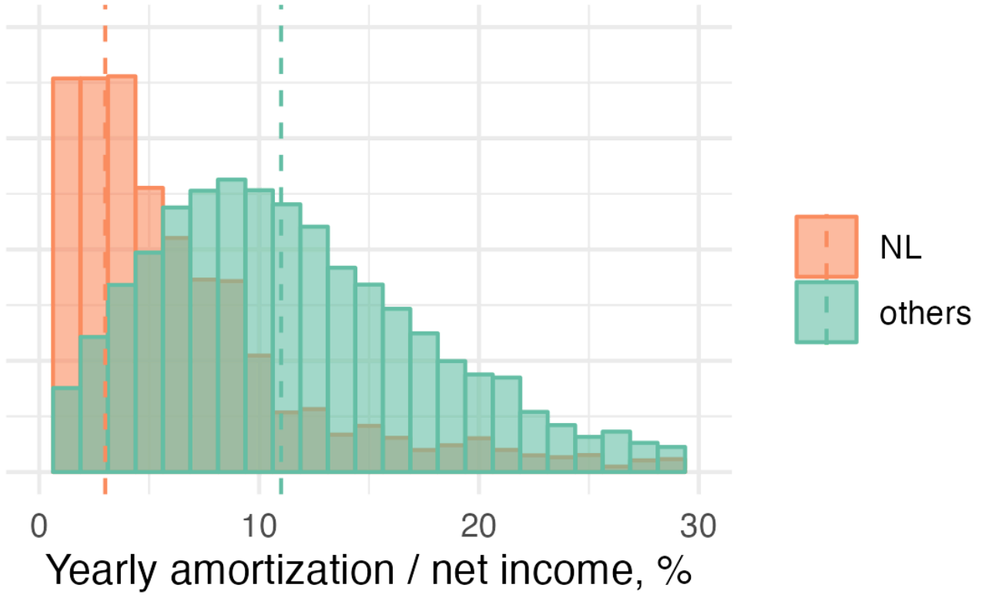
Appendix

Data: mortgage amortization in the HFCS

% of regular payment going to amortization



% of household income going to amortization



Dashed lines indicate medians.

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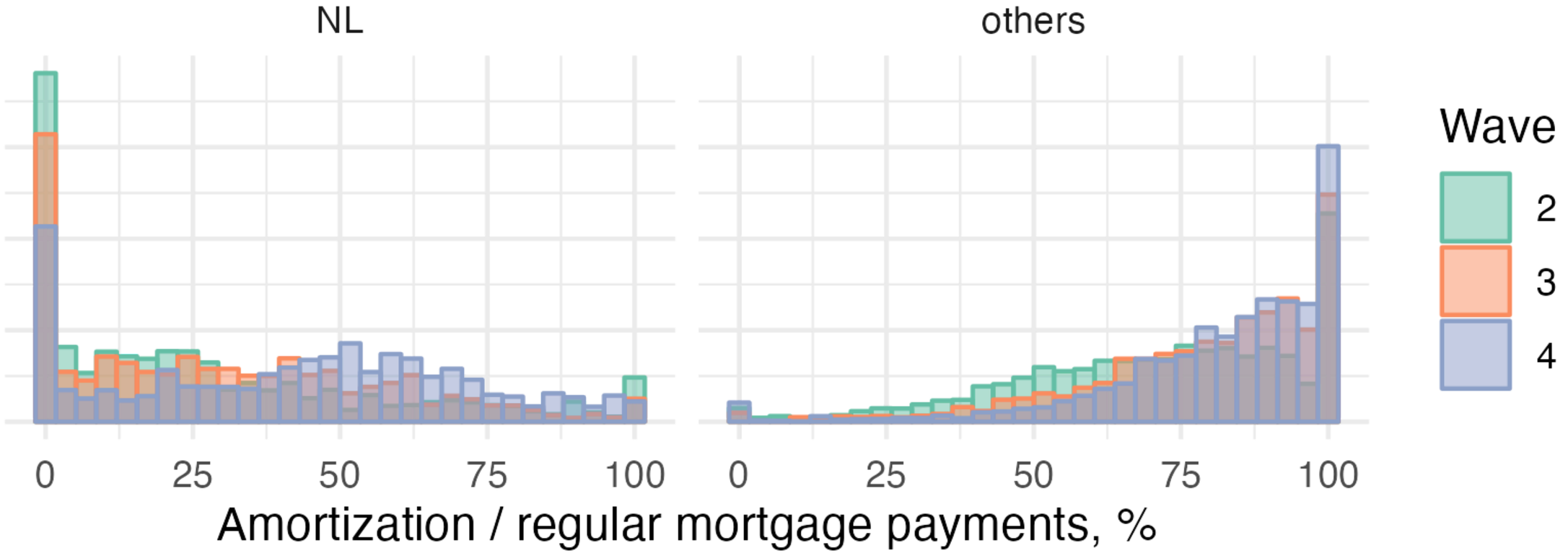
Appendix

Amortization by wave



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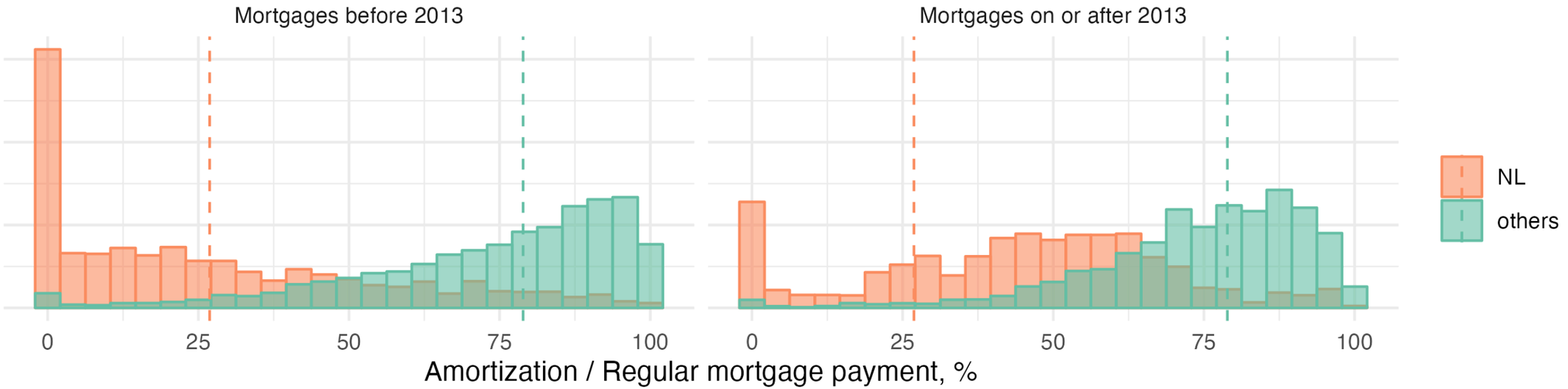
Appendix

Amortization for mortgages before and after 2013



Dashed lines indicate medians.

Percentage of obs. where amortization is less than 5% of the regular payment:

	NL	others
Mortgages before 2013	30.1	1.7
Mortgages on or after 2013	11.8	1.0

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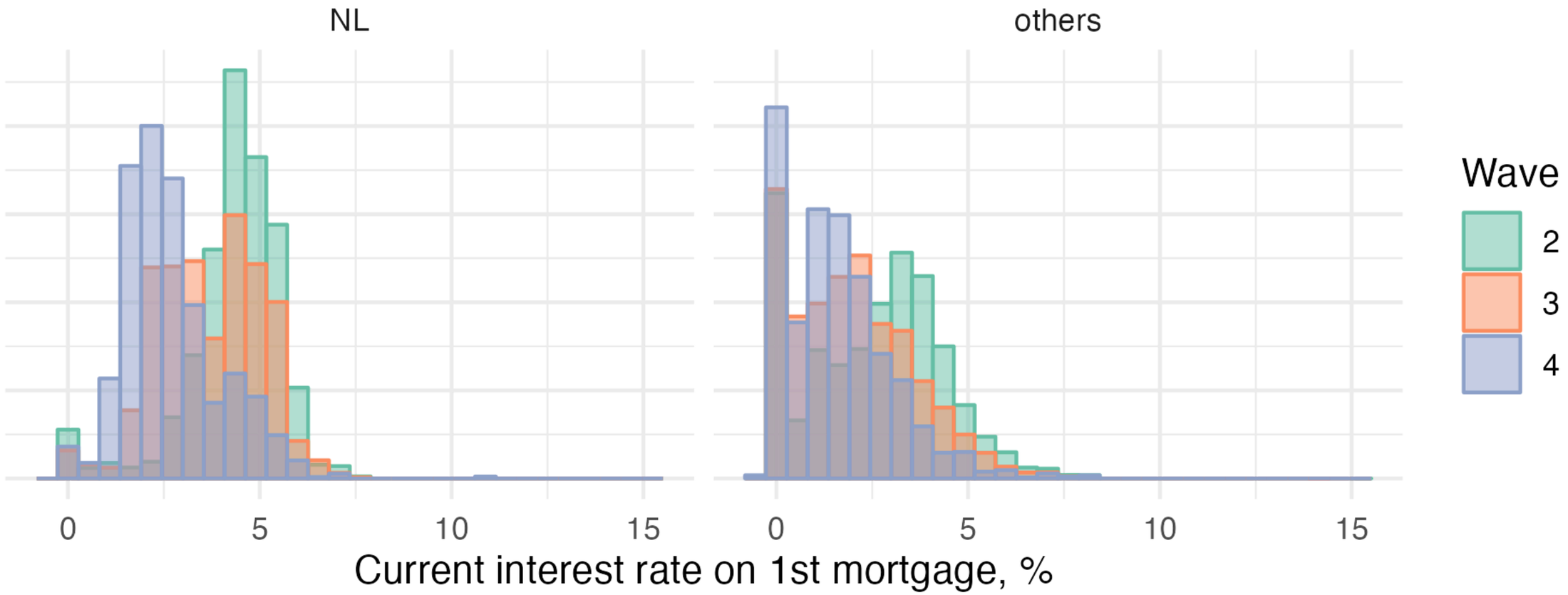
Appendix

Interest rates



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Appendix

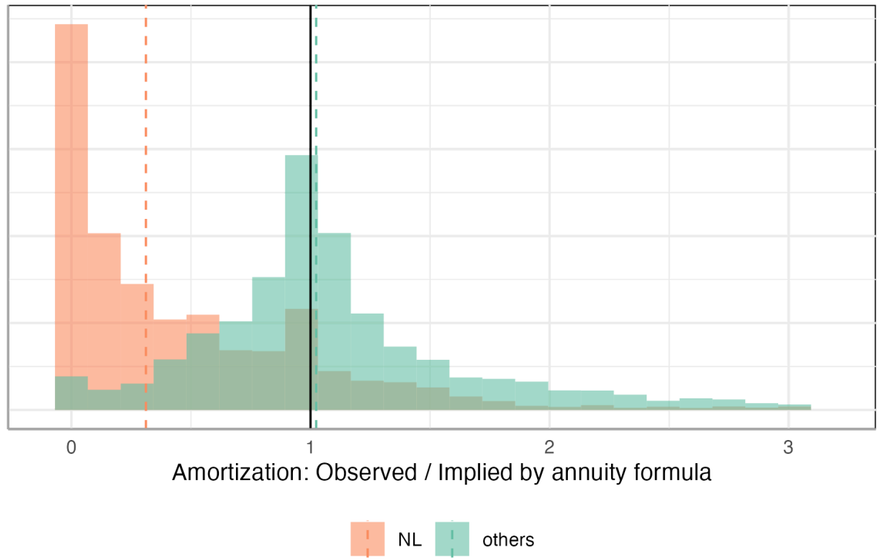
Amortization implied by annuity formula

- If mortgage is an annuitized loan, the amortization paid as part of the installment in period t is:

$$L \times r \times \left(\frac{1}{1 - \frac{1}{(1+r)^{T-t}}} - 1 \right)$$

→ where L is the outstanding amount, r the loan rate and T the residual maturity.

- This is what we observe for the median HH in the overall sample but not in NL:



Note: Dashed lines indicate country group medians.

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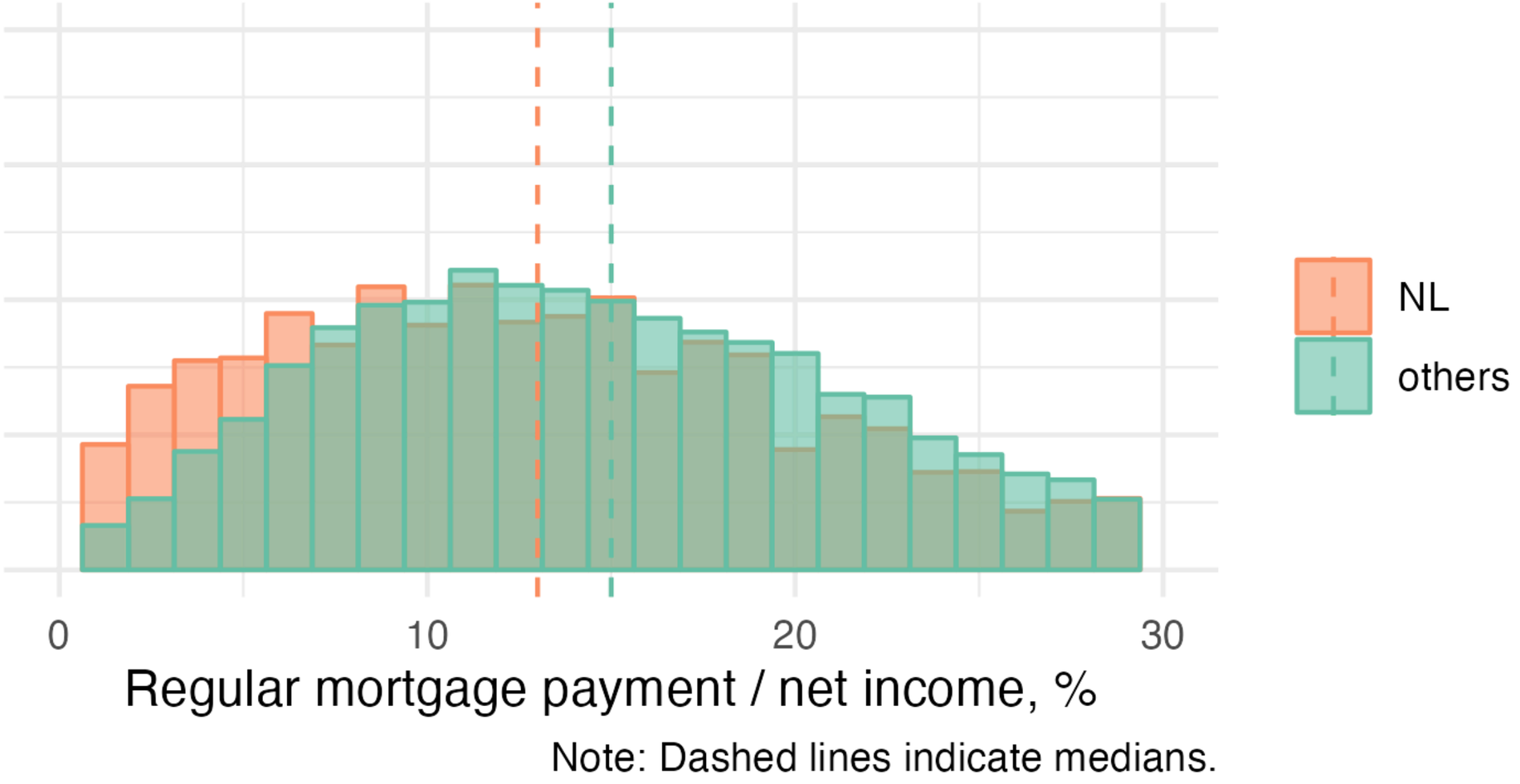
Appendix

Weight of regular mortgage payments on income



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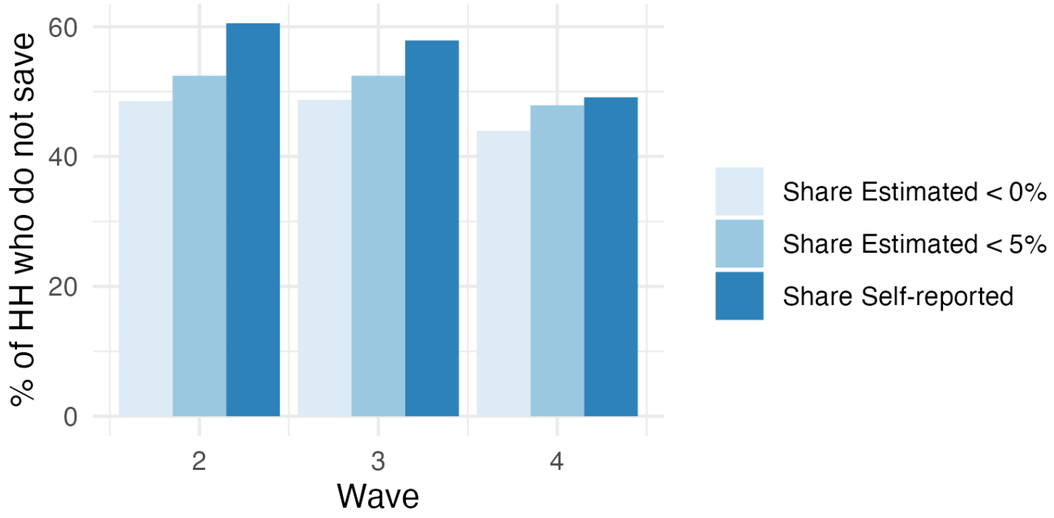
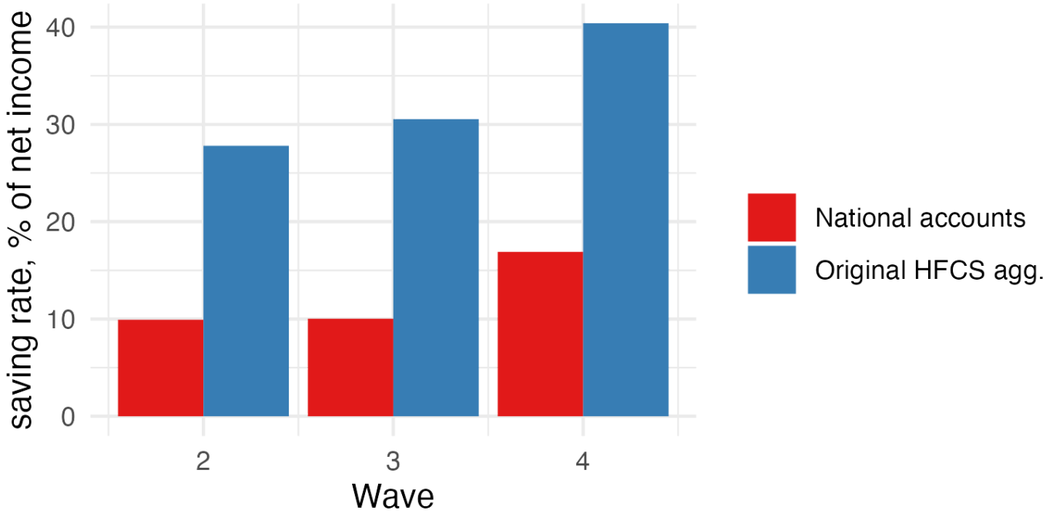
Appendix

Saving rate measure checks

- Match with self-reported ability to save:



- HFCS aggregates vs. national accounts (QSA)



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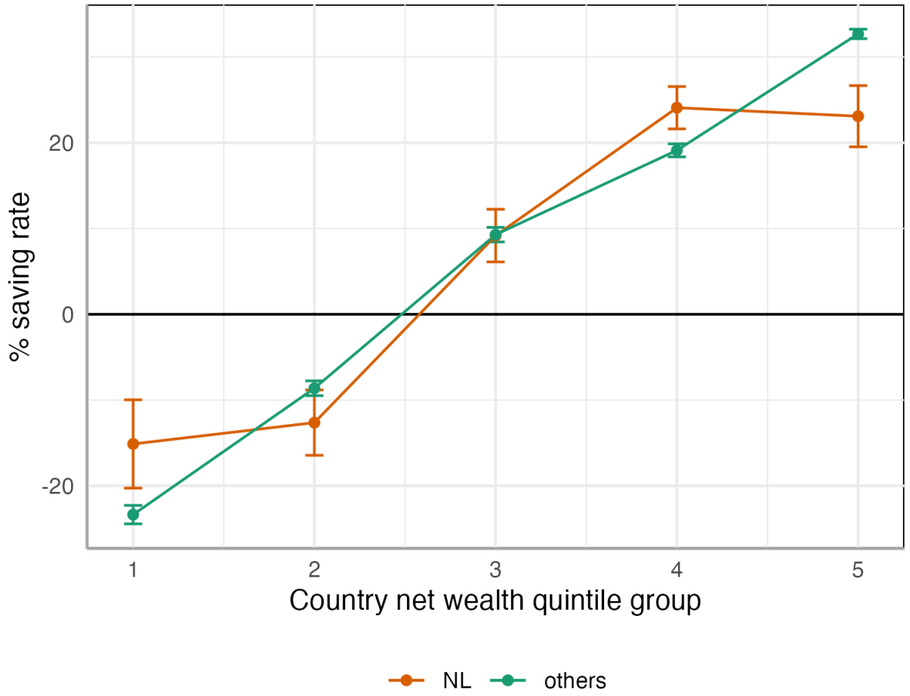
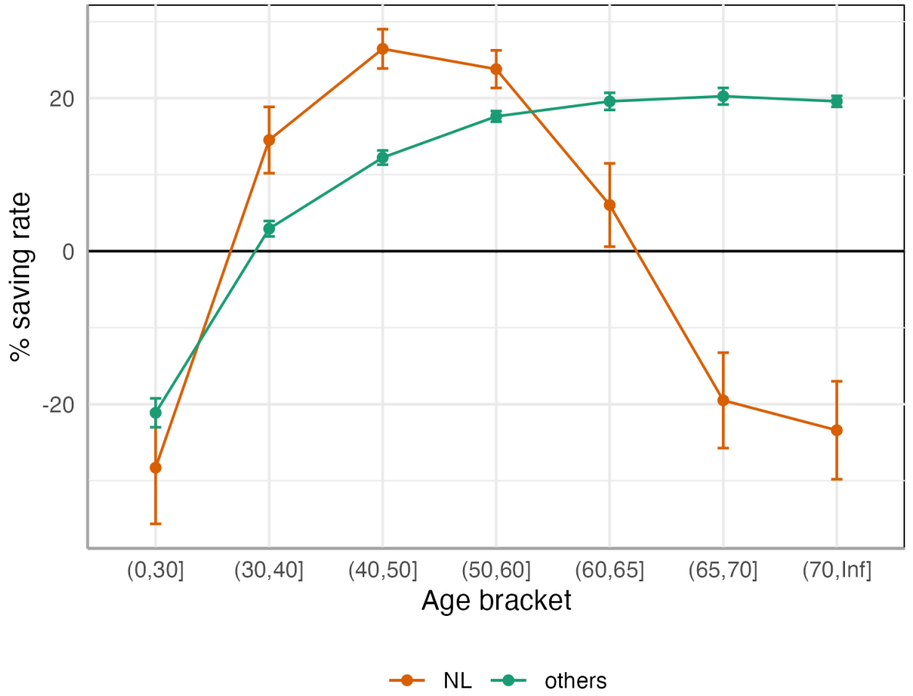
Appendix

Data: saving rates in the HFCS





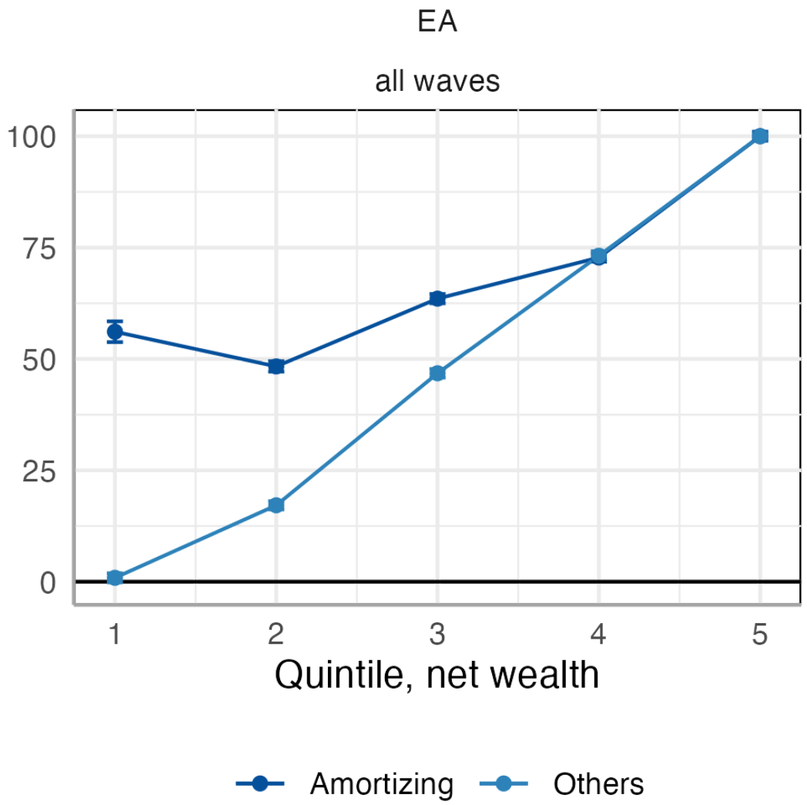
- Saving rates increase with wealth for both

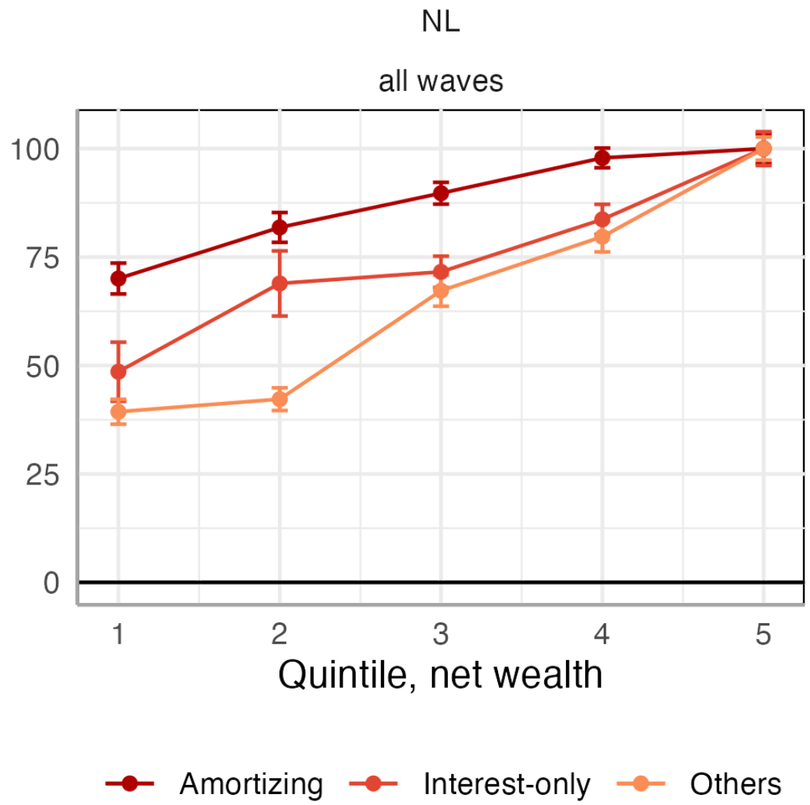
- Decline in old age in NL
- Interesting, as illiquidity of housing possible reason for plateau of saving (eg Yang 2009)

Appendix

Data from Euro area countries

Saving rates over the wealth distribution (Q5 = 100)





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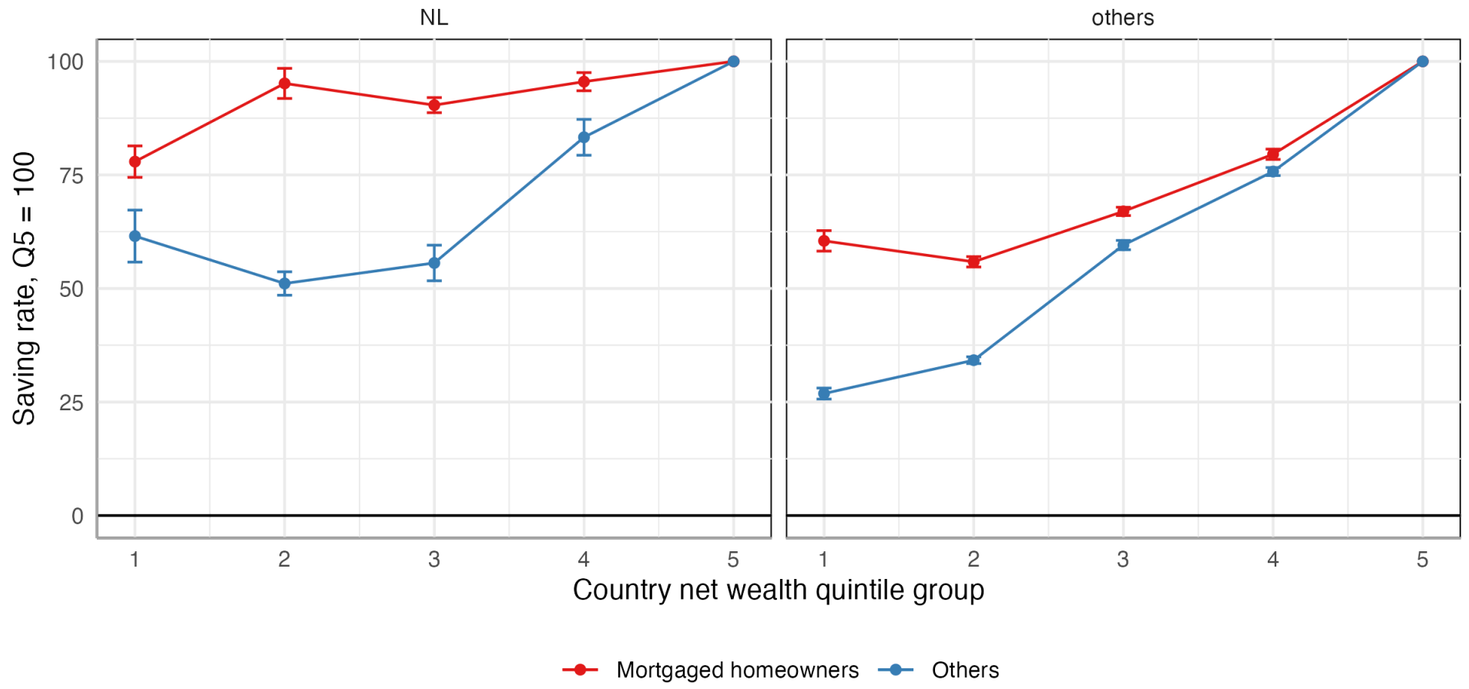
Appendix

Saving rates over the wealth distribution

Mortgaged homeowners vs. others

- Waves 3 and 4:



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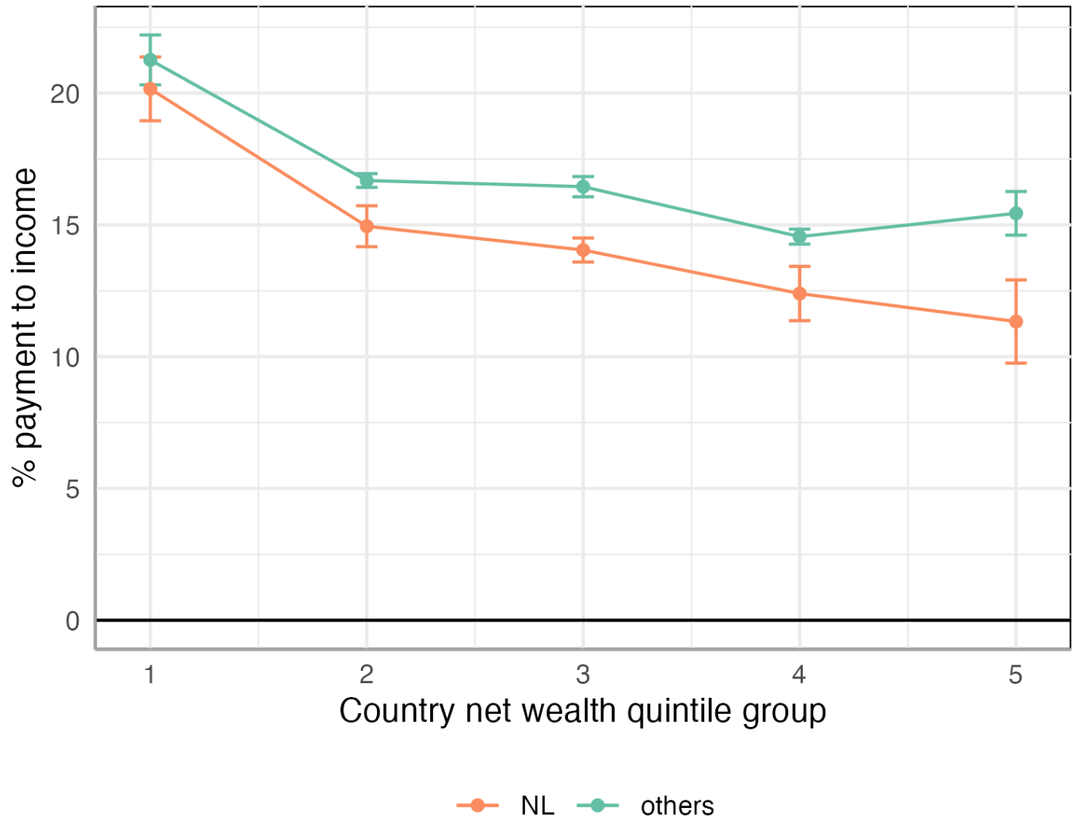
Appendix

Saving rates over the wealth distribution

Mortgaged homeowners vs. others



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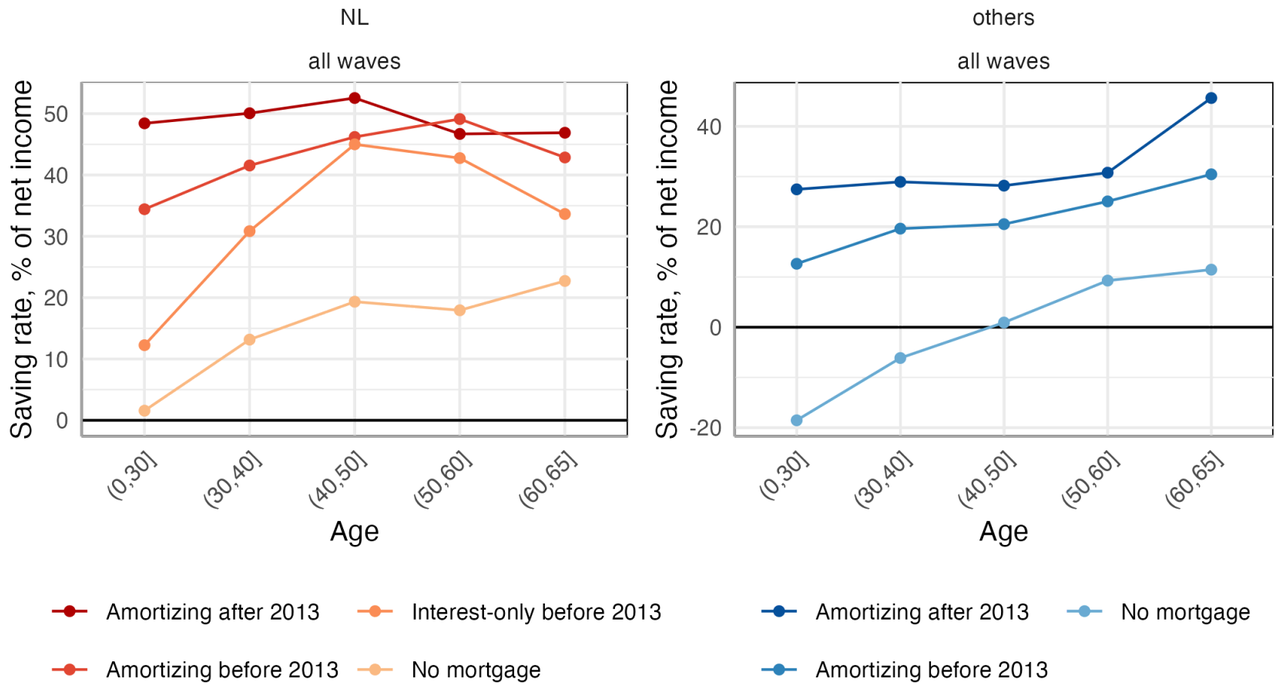
Appendix

Saving rates over the life cycle

Saving by homeowners in NL and others



- No substantial difference between post-policy reform mortgages

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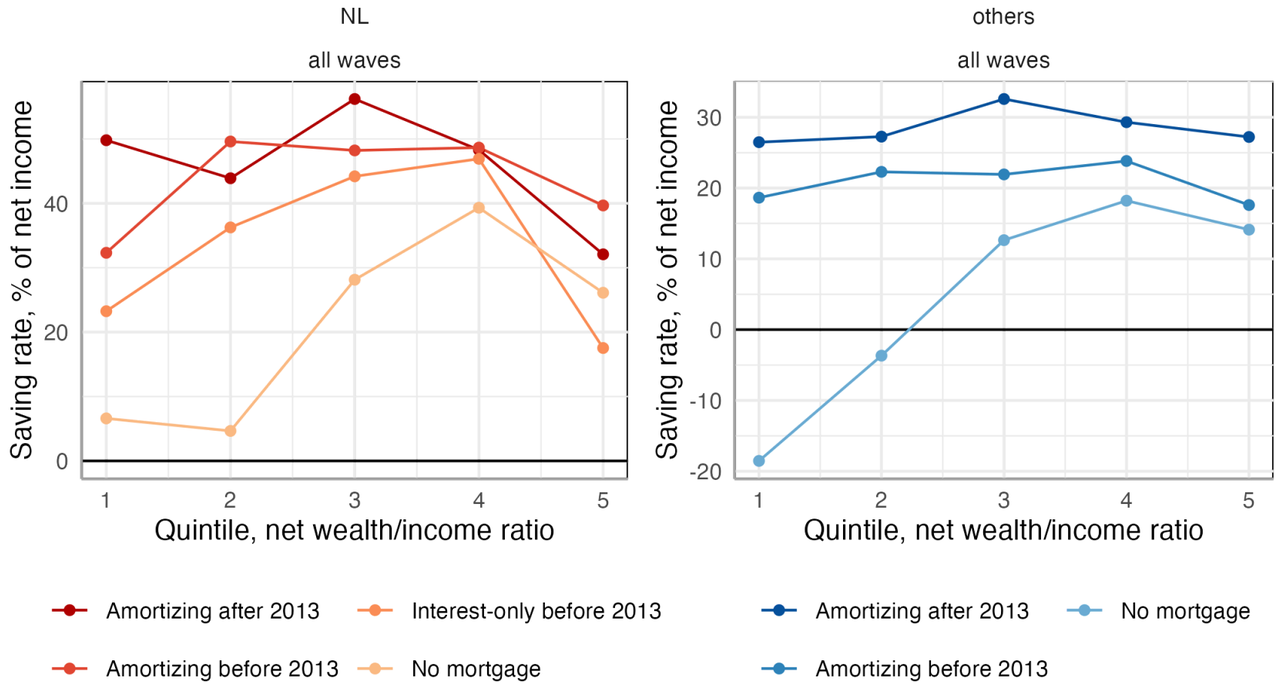
Appendix

Saving rates over the wealth distribution

Saving by homeowners in NL and others



- No substantial difference between post-policy reform mortgages

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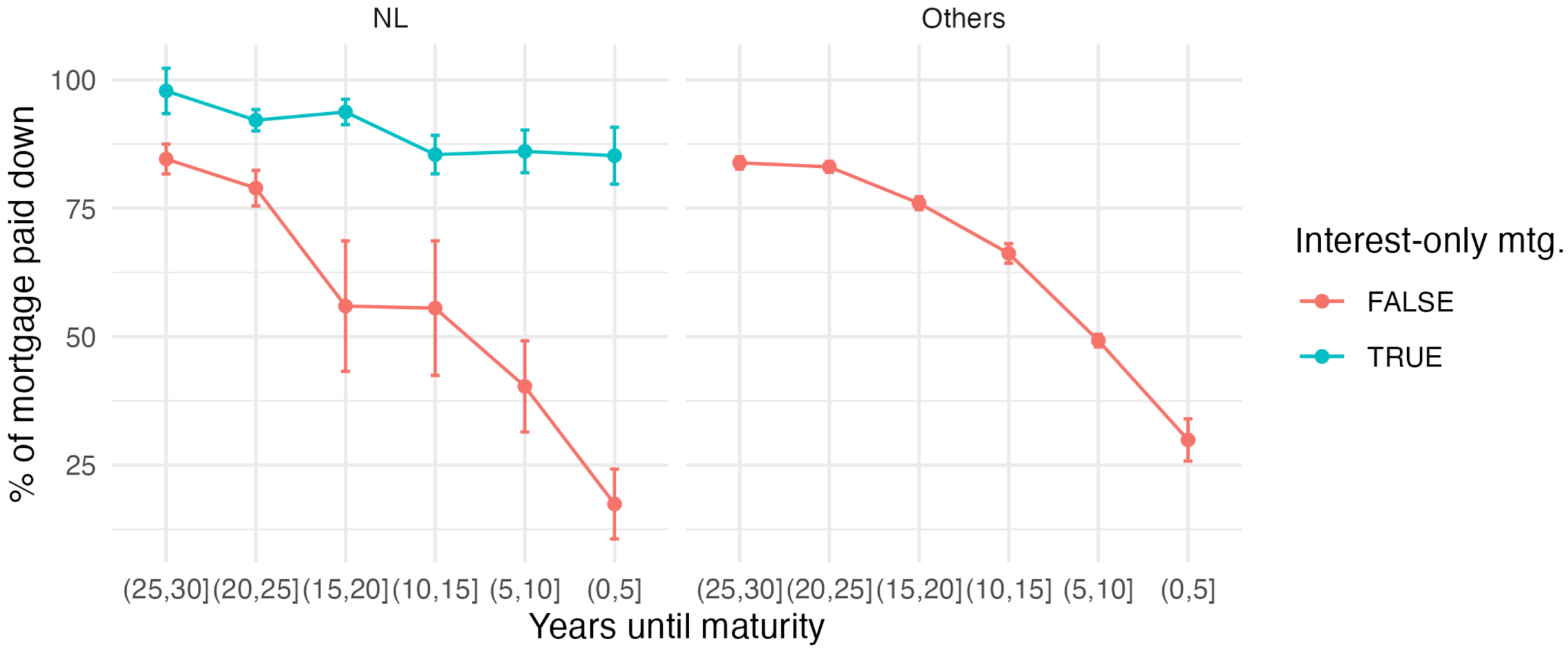
Appendix

Age profiles of debt in the data

- Life cycle profile of savings and mortgage debt
- Strict subsample of households who:
  - → Have never refinanced
  - → Live in their first home
    - ↦ Roughly identified by age at purchase ≤ 35
- Interest-only mortgages: those for which amortization is $<$ 80% implied by annuity formula

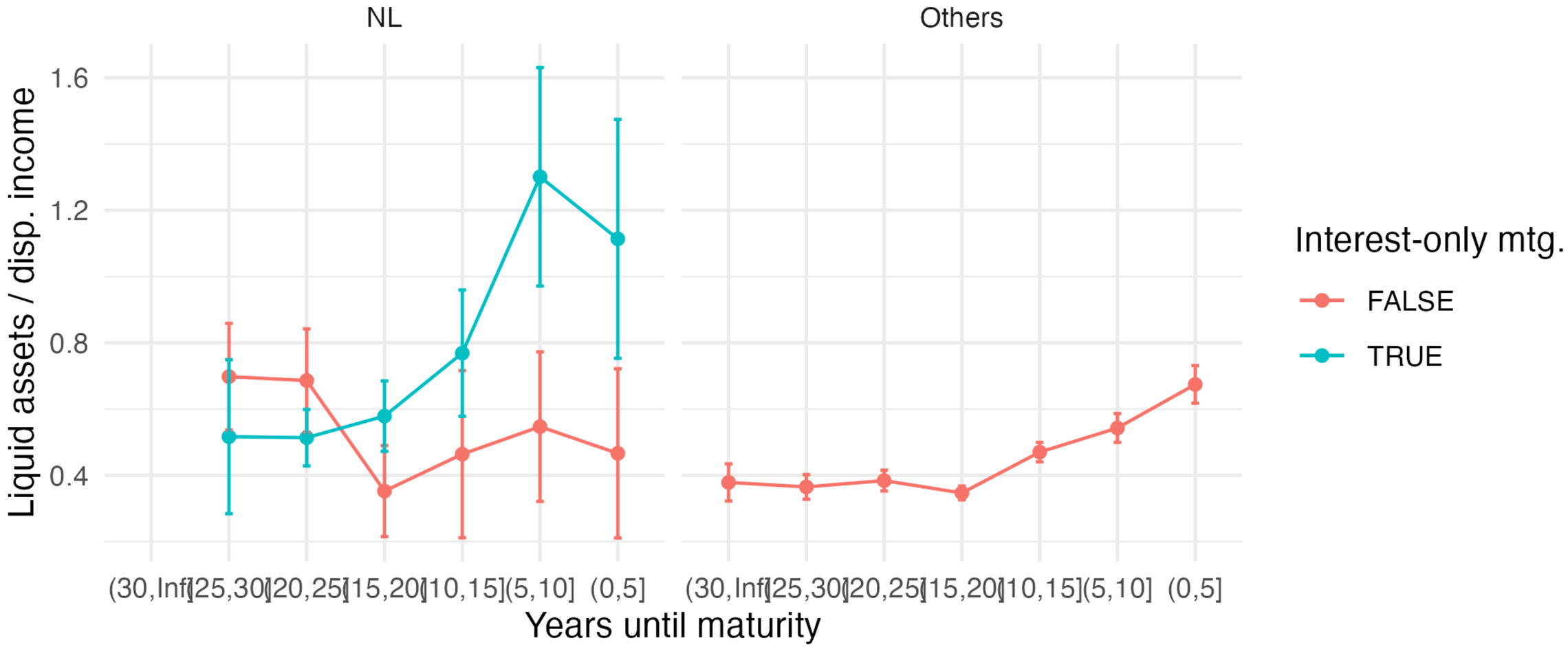
Appendix

Age profiles of debt in the data



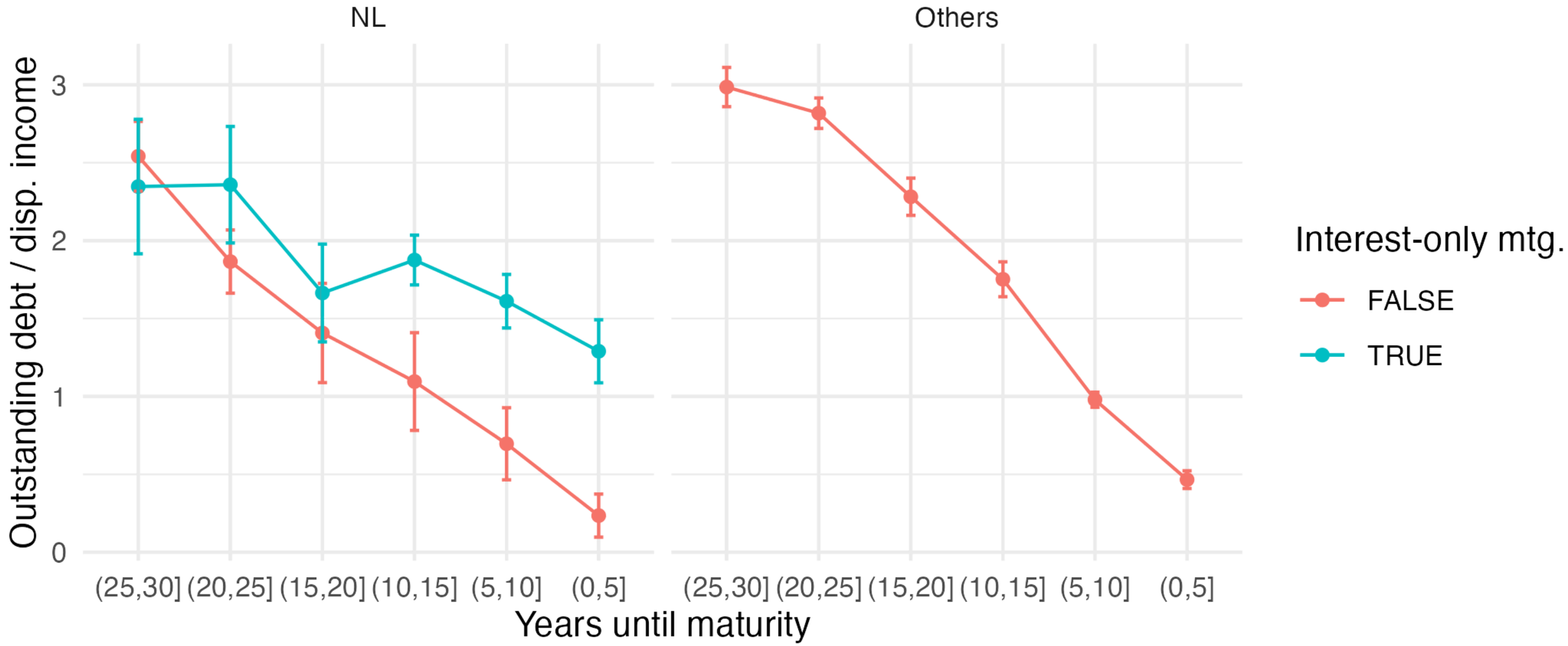
Appendix

Age profiles of debt in the data



Appendix

Age profiles of debt in the data



Appendix

Model solution details

HH problem and solution

Basic principle uses stochastic gradient descent to find parameters of neural network that solve for the optimal policy function.

- Machine learning techniques allow to compute the gradient $\nabla_\theta \tilde{V}\left(s_0, \theta; \hat{\pi}\right)$
  - → Computationally feasible with ML infrastructure, as neural networks are designed to work with problems with many dimension
  - → JAX-based solution (implemented by Barrera & Silva, 2024, **nndp**)
  - → Solved using Google Cloud TPU
- Adjust θ according to:

$$\Delta\theta = -\alpha \nabla_\theta \tilde{V}\left(s_0, \theta; \hat{\pi}\right)$$

  - → i.e., move in the direction that reduces the loss function ($-V$) the fastest
  - → α is the learning rate